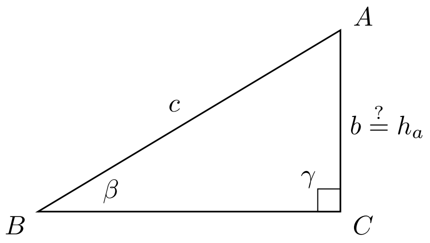
Page 68

	Data	Determine a triangle?	Restrictions?
1	ABa	Yes	$\angle A + \angle B < 180^\circ$
2	ABb	Yes	Same as 1
3	ABc	Yes	Same as 1
4	AbC	Yes	$\angle A + \angle C < 180^\circ$
5	ABC	No	-
6	Abc	Yes	None
7	Bbc	No	-
8	Cbc	No	-

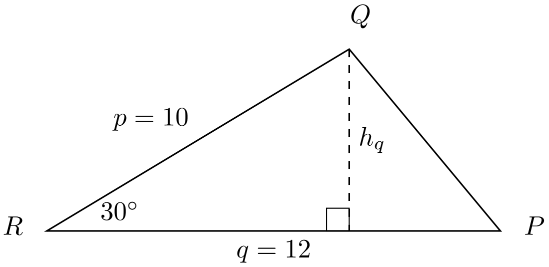
Page 70

3.



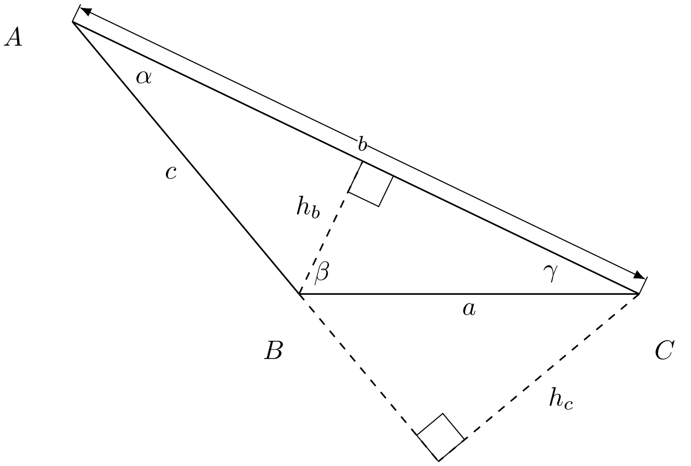
$$h_a = b \sin \gamma = b \sin 90^\circ = b.$$

4.



$$\text{Ar}(\triangle PQR) = \frac{1}{2} q h_q = \frac{1}{2} qp \sin 30^\circ = 30.$$

Page 71

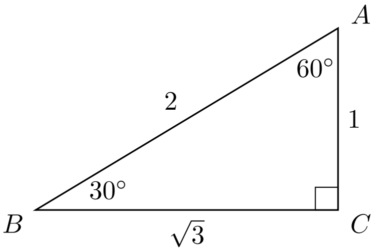


$$\begin{aligned} \sin\alpha &= h_b/c & \sin\gamma &= h_b/a \\ h_b &= c\sin\alpha & h_b &= a\sin\gamma \end{aligned}$$

$$\begin{aligned} \sin\alpha &= h_c/b & \sin(180^\circ-\beta) &= h_c/a \\ h_c &= b\sin\alpha & h_c &= a\sin(180^\circ-\beta) = a\sin\beta. \end{aligned}$$

Page 73

2.



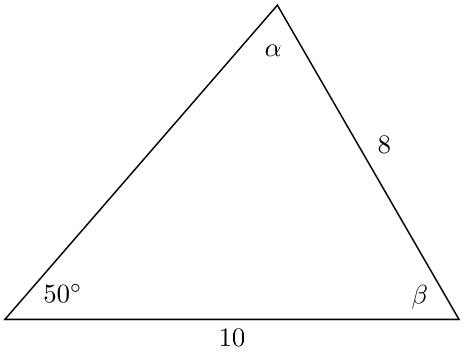
$$\begin{aligned} \frac{a}{\sin 60^\circ} &= \frac{\sqrt{3}}{\sqrt{3}/2} = 2, \\ \frac{b}{\sin 30^\circ} &= \frac{1}{1/2} = 2, \\ \frac{c}{\sin 90^\circ} &= \frac{2}{1} = 2. \end{aligned}$$

3. (a)
$$\frac{10}{\sin(180^\circ - (50^\circ + 70^\circ))} = \frac{x}{\sin 50^\circ} = \frac{y}{\sin 70^\circ}.$$
$$\therefore x = 10\left(\frac{\sin 50^\circ}{\sin 60^\circ}\right) \approx 8.85, \qquad y = 10\left(\frac{\sin 70^\circ}{\sin 60^\circ}\right) \approx 10.9.$$

(b)
$$\frac{10}{\sin(180^\circ - (65^\circ + 55^\circ))} = \frac{x}{\sin 55^\circ} = \frac{y}{\sin 65^\circ}.$$
$$\therefore x = 10\left(\frac{\sin 55^\circ}{\sin 60^\circ}\right) \approx 9.46, \qquad y = 10\left(\frac{\sin 65^\circ}{\sin 60^\circ}\right) \approx 10.5.$$

4. As the equations relating altitudes to sides and sines of angles were used in deriving the law of sines, and because these equations hold for obtuse-angled triangles (see Exercise 1 on Page 71), we can safely say that the law of sines holds for this type of triangle as well.

5.



$$\begin{aligned} \frac{8}{\sin 50^\circ} &= \frac{10}{\sin\alpha} \\ \sin\alpha &\approx 0.958 \\ \alpha &\approx \sin^{-1}(0.958) \approx 73.2^\circ. \\ \therefore \beta &\approx 180^\circ - (73.2^\circ + 50^\circ) = 56.8^\circ. \end{aligned}$$

6. $\sin 180^\circ - \theta$ has been defined to be equal to $\sin\theta$. Hence, in the previous answer, α could also be $180^\circ - 73.2^\circ = 106.8^\circ$ in which case β would be 23.2°.